$$\frac{d\alpha}{dh} = -\frac{dn}{dr} = \frac{2n_0 r}{r_0^{\,2}}$$

$$\frac{dr}{dh} = -\alpha$$

$$dh = d\alpha * \frac{r_0^{\,2}}{2n_0 r} = -\frac{dr}{\alpha}$$

$$\alpha * d\alpha = -\frac{dr}{r_0^{\,2}} 2n_0 r$$

$$\alpha^2 = \frac{R^2 - r^2}{r_0^{\,2}} 2n_0$$

$$r^2 = R^2 - \frac{\alpha^2 r_0^{\,2}}{2n_0}$$

$$\frac{d\alpha}{dh} = \frac{2n_0 r}{r_0^{\,2}} = \frac{2n_0}{r_0^{\,2}} \sqrt{R^2 - \frac{\alpha^2 r_0^{\,2}}{2n_0}}$$

$$\frac{d\alpha}{\sqrt{R^2 - \frac{\alpha^2 r_0^{\,2}}{2n_0}}} = \frac{2n_0}{r_0^{\,2}} dh$$

$$\alpha = \sin(\phi) \frac{R}{r_0} \sqrt{2n_0}$$

$$da = \cos(\phi) d\phi \frac{R}{r_0} \sqrt{2n_0}$$

$$\frac{d\phi \frac{R}{r_0} \sqrt{2n_0}}{R} = \frac{d\phi}{r_0} \sqrt{2n_0} = \frac{2n_0}{r_0^{\,2}} dh$$

$$\phi(h) = \sqrt{2n_0} \frac{h}{r_0}$$

$$\alpha(h) = \sin(\phi) \frac{R}{r_0} \sqrt{2n_0} = \sin\left(\sqrt{2n_0} \frac{h}{r_0} \right) * \frac{R}{r_0} \sqrt{2n_0}$$

$$\alpha(h = H) = \sin\left(\sqrt{2n_0} \frac{H}{r_0} \right) \frac{R}{r_0} \sqrt{2n_0}$$

$$\alpha \approx \frac{2n_0 HR}{r_0^{\,2}}$$

$$\beta = \alpha * n(r) = \alpha * n(r)\Big|_{r=\sqrt{R^2 - \frac{\alpha^2 r_0^{\,2}}{2n_0}}} = \alpha * n(r)\Big|_{r=\sqrt{R^2 - \frac{\left(\frac{2n_0 HR}{r_0^{\,2}}\right)^2 r_0^{\,2}}{2n_0}}} = \alpha * n(r)\Big|_{r=R\sqrt{1-2n_0\left(\frac{H}{r_0}\right)^2}} = \frac{2n_0 HR}{r_0^{\,2}} * n_0 \left(1 - \left(\frac{r}{r_0} \right)^2 \right)$$

$$= \frac{2n_0^{\,2} HR}{r_0^{\,2}} * \left(1 - \left(\frac{R\sqrt{1 - 2n_0 \left(\frac{H}{r_0} \right)^2}}{r_0} \right)^2 \right) = \frac{2n_0^{\,2} HR}{r_0^{\,2}} * \left(1 - \left(\frac{R}{r_0} \right)^2 \left(1 - 2n_0 \left(\frac{H}{r_0} \right)^2 \right) \right) = 0{,}044463 \text{ рад} = 2{,}54 \text{ град}$$