$\sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4}$

ОДЗ.

$$\begin{cases} 3x+4\geq 0 \\ x-3\geq 0 \\ 2x+1\geq 0 \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\begin{cases} x\geq -\frac{4}{3} \\ x-3\geq 0 \\ x\geq -\frac{1}{2} \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\begin{cases} x\geq -\frac{4}{3} \\ x-3\geq 0 \\ x\geq -\frac{1}{2} \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\begin{cases} x\geq -\frac{1}{2} \\ x-3\geq 0 \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\begin{cases} x\geq -\frac{1}{2} \\ x\geq 3 \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\begin{cases} x\geq 3 \\ \sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4} \end{cases}$$

$$\sqrt{2x+1}+\sqrt{x-3}=\sqrt{3x+4}$$

$$\left(\sqrt{2x+1}+\sqrt{x-3}\right)^2=3x+4$$

$$\sqrt{2x+1}^2+2\sqrt{2x+1}\sqrt{x-3}+\sqrt{x-3}^2=3x+4$$

$$(2x+1)+2\sqrt{2x+1}\sqrt{x-3}+(x-3)=3x+4$$

$$\begin{cases} x\geq 3 \\ (2x+1)+2\sqrt{2x+1}\sqrt{x-3}+(x-3)=3x+4 \end{cases}$$

$$\begin{cases} x\geq 3 \\ 2x+1+2\sqrt{2x+1}\sqrt{x-3}+x-3=3x+4 \end{cases}$$

$$\begin{cases} x\geq 3 \\ 2x-2+2\sqrt{2x+1}\sqrt{x-3}+x=3x+4 \end{cases}$$

$$\begin{cases} x\geq 3 \\ 3x-2+2\sqrt{2x+1}\sqrt{x-3}=3x+4 \end{cases}$$

$$3x-2+2\sqrt{(2x+1)(x-3)}=3x+4$$

$$\begin{cases} x\geq 3 \\ 3x-2+2\sqrt{(2x+1)(x-3)}=3x+4 \end{cases}$$

$$3x-2+2\sqrt{(2x+1)(x-3)}=3x+4$$

$$2\sqrt{(2x+1)(x-3)}=3x+4-3x+2$$

$$2\sqrt{(2x+1)(x-3)}=6+3x-3x$$

$$2\sqrt{(2x+1)(x-3)}=6$$

$$2^2((2x+1)(x-3))=6^2$$

$$\begin{cases} x\geq 3 \\ 2^2((2x+1)(x-3))=6^2 \end{cases}$$

$$\begin{cases} x\geq 3 \\ 2^2(2x+1)(x-3)=6^2 \end{cases}$$

$$\begin{cases} x\geq 3 \\ 4(2x+1)(x-3)=36 \end{cases}$$

$$\begin{cases} x\geq 3 \\ (2x+1)(x-3)=\frac{36}{4} \end{cases}$$

$$\begin{cases} x\geq 3 \\ (2x+1)(x-3)=9 \end{cases}$$

$$(2x+1)(x-3)=9$$

$$(2x+1)(x-3)-9=0$$

$$(2x^2-6x+x-3)-9=0$$

$$(2x^2-5x-3)-9=0$$

$$2x^2-5x-3-9=0$$

$$2x^2-5x-12=0$$

Находим дискриминант.

$$D=b^2-4ac=(-5)^2-4\cdot 2(-12)=121$$

$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$

$x_1=\frac{5-11}{2\cdot 2}=-\frac{3}{2}; x_2=\frac{5+11}{2\cdot 2}=4$

решение разбивается на отдельные случаи.

Случай 1 .

$\begin{cases} x\geq 3 \\ x=-\frac{3}{2} \end{cases}$

нет решений

Случай 2 .

$\begin{cases} x\geq 3 \\ x=4 \end{cases}$

Окончательный ответ: x=4

$2\sqrt{3x-2}-3\sqrt{2x-3}=\sqrt{3x-5}$

ОДЗ.

$\begin{cases} 3x-5\geq 0 \\ 2x-3\geq 0 \\ 3x-2\geq 0 \\ 2\sqrt{3x-2}-3\sqrt{2x-3}=\sqrt{3x-5} \end{cases}$

$\begin{cases} x\geq \frac{5}{3} \\ x\geq \frac{3}{2} \\ x\geq \frac{2}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}=\sqrt{3x-5} \end{cases}$

$\begin{cases} x\geq \frac{5}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}=\sqrt{3x-5} \end{cases}$

$2\sqrt{3x-2}-3\sqrt{2x-3}=\sqrt{3x-5}$

$\begin{cases} 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ (2\sqrt{3x-2}-3\sqrt{2x-3})^2=3x-5 \end{cases}$

$$\begin{cases}2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ \left(2\sqrt{3x-2}\right)^2-2\cdot 2\sqrt{3x-2}3\sqrt{2x-3}+\left(3\sqrt{2x-3}\right)^2=3x-5\end{cases}$$

$$\begin{cases}x\geq\frac{5}{3}\\ \begin{cases}2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ \left(2\sqrt{3x-2}\right)^2-2\cdot 2\sqrt{3x-2}3\sqrt{2x-3}+\left(3\sqrt{2x-3}\right)^2=3x-5\end{cases}\end{cases}$$

$$\begin{cases}x\geq\frac{5}{3}\\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ \left(2\sqrt{3x-2}\right)^2-12\sqrt{3x-2}\sqrt{2x-3}+\left(3\sqrt{2x-3}\right)^2=3x-5\end{cases}$$

$$\begin{cases}x\geq\frac{5}{3}\\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ 2^2(3x-2)-12\sqrt{3x-2}\sqrt{2x-3}+3^2(2x-3)=3x-5\end{cases}$$

$$\begin{cases}x\geq\frac{5}{3}\\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ 4(3x-2)-12\sqrt{3x-2}\sqrt{2x-3}+9(2x-3)=3x-5\end{cases}$$

$$4(3x-2)-12\sqrt{(3x-2)(2x-3)}+9(2x-3)=3x-5$$

$$\begin{cases}x\geq\frac{5}{3}\\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ 4(3x-2)-12\sqrt{(3x-2)(2x-3)}+9(2x-3)=3x-5\end{cases}$$

$$4(3x-2)-12\sqrt{(3x-2)(2x-3)}+9(2x-3)=3x-5$$

$$-12\sqrt{(3x-2)(2x-3)}=3x-5-4(3x-2)-9(2x-3)$$

$$12\sqrt{(3x-2)(2x-3)}=-3x+5+4(3x-2)+9(2x-3)$$

$$\begin{cases}-3x+5+4(3x-2)+9(2x-3)\geq 0\\ 12^2\left((3x-2)(2x-3)\right)=\left(-3x+5+4(3x-2)+9(2x-3)\right)^2\end{cases}$$

$$\begin{cases}x\geq\frac{5}{3}\\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0\\ \begin{cases}-3x+5+4(3x-2)+9(2x-3)\geq 0\\ 12^2\left((3x-2)(2x-3)\right)=\left(-3x+5+4(3x-2)+9(2x-3)\right)^2\end{cases}\end{cases}$$

$$\begin{cases} x\geq\frac{5}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ -3x+5+4(3x-2)+9(2x-3)\geq 0 \\ 12^2(3x-2)(2x-3)=(-3x+5+4(3x-2)+9(2x-3))^2 \end{cases}$$

$$\begin{cases} x\geq\frac{5}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ -3x+5+4(3x-2)+9(2x-3)\geq 0 \\ 144(3x-2)(2x-3)=(-3x+5+4(3x-2)+9(2x-3))^2 \end{cases}$$

$144(3x-2)(2x-3)=(-3x+5+4(3x-2)+9(2x-3))^2$

$144(3x-2)(2x-3)-(-3x+5+4(3x-2)+9(2x-3))^2=0$

$144(3x-2)(2x-3)-(-3x+5+(12x-8)+(18x-27))^2=0$

$144(3x-2)(2x-3)-(-3x+5+12x-8+18x-27)^2=0$

$144(3x-2)(2x-3)-(27x-30)^2=0$

$144(3x-2)(2x-3)-(729x^2-1620x+900)=0$

$(432x-288)(2x-3)-(729x^2-1620x+900)=0$

$(864x^2-1296x-576x+864)-(729x^2-1620x+900)=0$

$(864x^2-1872x+864)-(729x^2-1620x+900)=0$

$864x^2-1872x+864-729x^2+1620x-900=0$

$135x^2-252x-36=0$

$15x^2-28x-4=0$

Находим дискриминант.

$D=b^2-4ac=(-28)^2-4\cdot 15(-4)=1024$

$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$

$x_1=\frac{28-32}{2\cdot 15}=-\frac{2}{15}; x_2=\frac{28+32}{2\cdot 15}=2$

решение разбивается на отдельные случаи.

Случай 1 .

$$\begin{cases} x\geq\frac{5}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ -3x+5+4(3x-2)+9(2x-3)\geq 0 \\ x=-\frac{2}{15} \end{cases}$$

нет решений

Случай 2 .

$$\begin{cases} x\geq\frac{5}{3} \\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ -3x+5+4(3x-2)+9(2x-3)\geq 0 \\ x=2 \end{cases}$$

$$\begin{cases} x=2 \\ 2\sqrt{3x-2}-3\sqrt{2x-3}\geq 0 \\ -3x+5+4(3x-2)+9(2x-3)\geq 0 \end{cases}$$

Подставим вместо x .

$$\begin{cases} x=2 \\ 2\sqrt{3\cdot 2-2}-3\sqrt{2\cdot 2-3}\geq 0 \\ -3\cdot 2+5+4(3\cdot 2-2)+9(2\cdot 2-3)\geq 0 \end{cases}$$

$$\begin{cases} x=2 \\ 1\geq 0 \\ 24\geq 0 \end{cases}$$

ответ:x[=2

$\sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24}$

ОДЗ.

$$\begin{cases} 25x+24\geq 0 \\ 9x+7\geq 0 \\ 4x+5\geq 0 \\ \sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24} \end{cases}$$

$$\begin{cases} x\geq-\frac{24}{25} \\ x\geq-\frac{7}{9} \\ x\geq-\frac{5}{4} \\ \sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24} \end{cases}$$

$$\begin{cases} x\geq -\frac{24}{25} \\ x\geq -\frac{7}{9} \\ x\geq -\frac{5}{4} \\ \sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24} \end{cases}$$

$$\begin{cases} x\geq -\frac{7}{9} \\ \sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24} \end{cases}$$

$$\sqrt{4x+5}+\sqrt{9x+7}=\sqrt{25x+24}$$

$$\left(\sqrt{4x+5}+\sqrt{9x+7}\right)^2=25x+24$$

$$\sqrt{4x+5}^2+2\sqrt{4x+5}\sqrt{9x+7}+\sqrt{9x+7}^2=25x+24$$

$$(4x+5)+2\sqrt{4x+5}\sqrt{9x+7}+(9x+7)=25x+24$$

$$\begin{cases} x\geq -\frac{7}{9} \\ (4x+5)+2\sqrt{4x+5}\sqrt{9x+7}+(9x+7)=25x+24 \end{cases}$$

$$\begin{cases} x\geq -\frac{7}{9} \\ 4x+5+2\sqrt{4x+5}\sqrt{9x+7}+9x+7=25x+24 \end{cases}$$

$$\begin{cases} x\geq -\frac{7}{9} \\ 4x+12+2\sqrt{4x+5}\sqrt{9x+7}+9x=25x+24 \end{cases}$$

$$\begin{cases} x\geq -\frac{7}{9} \\ 13x+12+2\sqrt{4x+5}\sqrt{9x+7}=25x+24 \end{cases}$$

$$13x+12+2\sqrt{(4x+5)(9x+7)}=25x+24$$

$$\begin{cases} x\geq -\frac{7}{9} \\ 13x+12+2\sqrt{(4x+5)(9x+7)}=25x+24 \end{cases}$$

$$13x+12+2\sqrt{(4x+5)(9x+7)}=25x+24$$

$$2\sqrt{(4x+5)(9x+7)}=25x+24-13x-12$$

$$2\sqrt{(4x+5)(9x+7)}=12+25x-13x$$

$$2\sqrt{(4x+5)(9x+7)}=12+12x$$

$$\begin{cases}12+12x\geq 0\\ 22\left((4x+5)(9x+7)\right)=(12+12x)^2\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ \begin{cases}12+12x\geq 0\\ 22\left((4x+5)(9x+7)\right)=(12+12x)^2\end{cases}\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ 12+12x\geq 0\\ 22(4x+5)(9x+7)=(12+12x)^2\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ 12+12x\geq 0\\ 4(4x+5)(9x+7)=(12+12x)^2\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ 1+x\geq 0\\ 4(4x+5)(9x+7)=(12+12x)^2\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ x\geq -1\\ 4(4x+5)(9x+7)=(12+12x)^2\end{cases}$$

$$\begin{cases}x\geq -\frac{7}{9}\\ 4(4x+5)(9x+7)=(12+12x)^2\end{cases}$$

$$4(4x+5)(9x+7)=(12+12x)^2$$

$$4(4x+5)(9x+7)-(12+12x)^2=0$$

$$4(4x+5)(9x+7)-(12x+12)^2=0$$

$$4(4x+5)(9x+7)-\left(144x^2+288x+144\right)=0$$

$$(16x+20)(9x+7)-\left(144x^2+288x+144\right)=0$$

$$\left(144x^2+112x+180x+140\right)-\left(144x^2+288x+144\right)=0$$

$$\left(144x^2+292x+140\right)-\left(144x^2+288x+144\right)=0$$

$$144x^2+292x+140-144x^2-288x-144=0$$

$$4x-4=0$$

$$4x=4$$

$x=4:4$

$x=1$

$$\begin{cases} x\geq -\frac{7}{9} \\ x=1 \end{cases}$$

Подставим вместо x .

$$\begin{cases} 1\geq -\frac{7}{9} \\ x=1 \end{cases}$$

Окончательный ответ: x=1