

$$\begin{aligned}
\frac{x-1}{x-2} - \frac{x+1}{3x+1} \cdot \frac{9x^2}{x^2-x-2} &= \frac{x-1}{x-2} - \frac{(x+1)9x^2}{(3x+1)(x^2-x-2)} = \frac{x-1}{x-2} - \frac{(x+1)9x^2}{(3x+1)(x-2)(x+1)} = \\
&= \frac{x-1}{x-2} - \frac{9x^2}{(3x+1)(x-2)} = \frac{(x-1)(3x+1)}{(x-2)(3x+1)} - \frac{9x^2}{(3x+1)(x-2)} = \frac{(x-1)(3x+1)-9x^2}{(x-2)(3x+1)} = \\
&= \frac{(3x^2+x-3x-1)-9x^2}{(x-2)(3x+1)} = \frac{(3x^2-2x-1)-9x^2}{(x-2)(3x+1)} = \frac{3x^2-2x-1-9x^2}{(x-2)(3x+1)} = \frac{-6x^2-2x-1}{(x-2)(3x+1)} = \\
&= -\frac{6x^2+2x+1}{(x-2)(3x+1)}
\end{aligned}$$

Если нужно решить по действиям, тогда так:

1)

$$\frac{x+1}{3x+1} \cdot \frac{9x^2}{x^2-x-2} = \frac{(x+1)9x^2}{(3x+1)(x^2-x-2)} = \frac{(x+1)9x^2}{(3x+1)(x-2)(x+1)} = \frac{9x^2}{(3x+1)(x-2)}$$

2)

$$\begin{aligned}
\frac{x-1}{x-2} - \frac{9x^2}{(3x+1)(x-2)} &= \frac{(x-1)(3x+1)}{(x-2)(3x+1)} - \frac{9x^2}{(3x+1)(x-2)} = \frac{(x-1)(3x+1)-9x^2}{(x-2)(3x+1)} = \\
&= \frac{(3x^2+x-3x-1)-9x^2}{(x-2)(3x+1)} = \frac{(3x^2-2x-1)-9x^2}{(x-2)(3x+1)} = \frac{3x^2-2x-1-9x^2}{(x-2)(3x+1)} = \frac{-6x^2-2x-1}{(x-2)(3x+1)} = \\
&= -\frac{6x^2+2x+1}{(x-2)(3x+1)}
\end{aligned}$$