$$\frac{3x+1}{x+2}-\frac{x-1}{x-2}=1$$

ОДЗ.

$$\begin{cases} x+2\neq 0\ (1) \\ x-2\neq 0\ (2) \end{cases}$$

$$\frac{3x+1}{x+2}-\frac{x-1}{x-2}-1=0$$

$$-1+\frac{3x+1}{x+2}-\frac{x-1}{x-2}=0$$

$$-1+\frac{(3x+1)(x-2)}{(x+2)(x-2)}-\frac{(x-1)(x+2)}{(x-2)(x+2)}=0$$

$$-1+\frac{(3x+1)(x-2)-(x-1)(x+2)}{(x+2)(x-2)}=0$$

$$-1+\frac{\left(3x^2-6x+x-2\right)-\left(x^2+2x-x-2\right)}{(x+2)(x-2)}=0$$

$$-1+\frac{\left(3x^2-5x-2\right)-\left(x^2+x-2\right)}{(x+2)(x-2)}=0$$

$$-1+\frac{3x^2-5x-2-x^2-x+2}{(x+2)(x-2)}=0$$

$$-1+\frac{2x^2-6x}{(x+2)(x-2)}=0$$

$$-\frac{(x+2)(x-2)}{(x+2)(x-2)}+\frac{2x^2-6x}{(x+2)(x-2)}=0$$

$$\frac{-(x+2)(x-2)+\left(2x^2-6x\right)}{(x+2)(x-2)}=0$$

$$\frac{-\left(x^2-4\right)+\left(2x^2-6x\right)}{(x+2)(x-2)}=0$$

$$\frac{-x^2+4+2x^2-6x}{(x+2)(x-2)}=0$$

$$\frac{x^2+4-6x}{(x+2)(x-2)}=0$$

$$\frac{x^2-6x+4}{(x+2)(x-2)}=0$$

$$x^2-6x+4=0$$

$$D=b^2-4ac=(-6)^2-4\cdot 1\cdot 4=20$$

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$

$$x_1=\frac{6-2\sqrt{5}}{2\cdot 1}=3-\sqrt{5}\ ;x_2=\frac{6+2\sqrt{5}}{2\cdot 1}=3+\sqrt{5}$$

Ответ : $x=3-\sqrt{5};x=3+\sqrt{5}$.