*Дана функция* y(x) = x3 –12x - 3.

1) Область определения функции. Так как функция не имеет дроби или корня, то нет ограничения в области её определения.

D(y) = (−∞; +∞).

2) Четность и нечетность функции:

Проверим функцию - четна или нечетна с помощью соотношений f(x)=f(-x) и f(x)=-f(x). Итак, проверяем: $f\left(-x\right)=\left(-x\right)^{3}-12\*\left(-x\right)-3=-x^{3}+12x-3\ne f\left(x\right)\ne -f\left(x\right).$

3начит, функция не является ни чётной, ни нечётной.

3) Определим точки пересечения графика функции с осями координат.

Найдем точки пересечения с осью ординат Oy, для чего приравниваем x = 0: у = 03 – 12\*0 - 3 = -3.

Таким образом, точка пересечения с осью Oy имеет координаты (0;3).

Найдем точки пересечения с осью абсцисс Ox, для чего надо решить кубическое уравнение x3 – 12x - 3 = 0.

Для вычисления корней этого кубического уравнения используем тригонометрическую формулу Виета, которая работает для уравнений вида:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| x | 3 |

 | + | a |

|  |  |
| --- | --- |
| x | 2 |

 | + | bx | + | c | = | 0. |

Если уравнение не такого вида, то его можно получить поделив всё уравнение на коэффициент возле

|  |  |  |  |
| --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | 3 |

 |  |

. В нашем случае

|  |  |  |
| --- | --- | --- |
| *a* | = | 0 |

,

|  |  |  |
| --- | --- | --- |
| *b* | = | −12 |

 и

|  |  |  |
| --- | --- | --- |
| *c* | = | −3 |

.
Теперь использовав формулы:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Q* | = |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|

|  |  |  |  |
| --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *a* | 2 |

 | −3*b* |

 |
| 9 |

 |  |

 и

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *R* | = |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 2 |

|  |  |
| --- | --- |
| *a* | 3 |

 | −9*ab* | + | 27*c* |

 |
| 54 |

 |  |

 вычислим, что

|  |  |  |
| --- | --- | --- |
| *Q* | = | 4 |

 и

|  |  |  |
| --- | --- | --- |
| *R* | = | −1.5 |

.
Далее по формуле

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *S* | = |

|  |  |
| --- | --- |
| *Q* | 3 |

 | − |

|  |  |
| --- | --- |
| *R* | 2 |

 |  |

 видим, что

|  |  |  |
| --- | --- | --- |
| *S* | > | 0, |

поэтому уравнение будет иметь три вещественных корня.
Которые вычисляются по следующим формулам:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | 1 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | *Q* |

 | cos |

|  |  |  |
| --- | --- | --- |
| ( | *ψ* | ) |

 | − |

|  |
| --- |
| *a* |
| 3 |

 |

 |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | 2 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | *Q* |

 | cos |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ( | *ψ* | + |

|  |
| --- |
| 2 |
| 3 |

 | *π* | ) |

 | − |

|  |
| --- |
| *a* |
| 3 |

 |

 |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | 3 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | *Q* |

 | cos |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ( | *ψ* | − |

|  |
| --- |
| 2 |
| 3 |

 | *π* | ) |

 | − |

|  |
| --- |
| *a* |
| 3 |

 |

 |

 |
| где *ψ* | = |

|  |
| --- |
| 1 |
| 3 |

 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (arccos |

|  |
| --- |
| *R* |
|

|  |  |  |  |
| --- | --- | --- | --- |
| √ |

|  |  |
| --- | --- |
| *Q* | 3 |

 |

 |

 | ) |

 |

 |

.
Подставив наши числа в эти формулы, мы получим:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | 1 |

 | = | −3.3316 | ; |

|  |  |
| --- | --- |
| *x* | 2 |

 | = | 3.5829 | ; |

|  |  |
| --- | --- |
| *x* | 3 |

 | = | −0.2513. |

4) Стационарные точки , интервалы возрастания и убывания функции , экстремумы функции

Исследуем функцию на экстремумы и монотонность. Для этого найдем первую производную функции: y’ = (x3 – 12x - 3)’ = 3x2 – 12 = 3(x2 – 4).

Приравняем первую производную к нулю и найдем стационарные точки (в которых y′=0): 3(x2 – 4) = 0, x = ±2.

Получили две критических точки:  х = -2 и х = 2.

Разобьем всю область определения функции на интервалы данными точками и определим знаки производной в каждом промежутке:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x = | -3 | -2 | 0 | 2 | 3 |
| y' = | 15 | 0 | -12 | 0 | 15 |

При x ∈ (−2; 2) производная y′ < 0, поэтому функция убывает на данном промежутке.

При x ∈ (-∞; -2) U (2; ∞) производная y′ > 0, функция возрастает на данных промежутках. При этом x = -2 - точка локального максимума (функция возрастает, а потом убывает, x = 2 - точка локального минимума (функция убывает, а потом возрастает.

Значение функции в этих точках: у(-1) = 13, у(1) = -19.

5) Дополнительные точки для построения графика функции y(x) = x3 − 12x - 3:

|  |  |
| --- | --- |
| **x** | **y** |
| -4.0 | -19 |
| -3.5 | -3.9 |
| -3.0 | 6 |
| -2.5 | 11.4 |
| -2.0 | 13 |
| -1.5 | 11.6 |
| -1.0 | 8 |
| -0.5 | 2.9 |
| 0 | -3 |
| 0.5 | -8.9 |
| 1.0 | -14 |
| 1.5 | -17.6 |
| 2.0 | -19 |
| 2.5 | -17.4 |
| 3.0 | -12 |
| 3.5 | -2.1 |
| 4.0 | 13 |
| 4.5 | 34.1 |

6) По полученным данным строим график, и отметим характерные точки (пересечения с осями и экстремумы).

