*Дана функция* y(x) = x3 –12x - 3.

1) Область определения функции. Так как функция не имеет дроби или корня, то нет ограничения в области её определения.

D(y) = (−∞; +∞).

2) Четность и нечетность функции:

Проверим функцию - четна или нечетна с помощью соотношений f(x)=f(-x) и f(x)=-f(x). Итак, проверяем:

3начит, функция не является ни чётной, ни нечётной.

3) Определим точки пересечения графика функции с осями координат.

Найдем точки пересечения с осью ординат Oy, для чего приравниваем x = 0: у = 03 – 12\*0 - 3 = -3.

Таким образом, точка пересечения с осью Oy имеет координаты (0;3).

Найдем точки пересечения с осью абсцисс Ox, для чего надо решить кубическое уравнение x3 – 12x - 3 = 0.

Для вычисления корней этого кубического уравнения используем тригонометрическую формулу Виета, которая работает для уравнений вида:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | x | 3 | | + | a | |  |  | | --- | --- | | x | 2 | | + | bx | + | c | = | 0. |

Если уравнение не такого вида, то его можно получить поделив всё уравнение на коэффициент возле

|  |  |  |  |
| --- | --- | --- | --- |
| |  |  | | --- | --- | | *x* | 3 | |  |

. В нашем случае

|  |  |  |
| --- | --- | --- |
| *a* | = | 0 |

,

|  |  |  |
| --- | --- | --- |
| *b* | = | −12 |

 и

|  |  |  |
| --- | --- | --- |
| *c* | = | −3 |

.  
Теперь использовав формулы:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Q* | = | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | |  |  | | --- | --- | | *a* | 2 | | −3*b* | | | 9 | |  |

 и

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *R* | = | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 2 | |  |  | | --- | --- | | *a* | 3 | | −9*ab* | + | 27*c* | | | 54 | |  |

 вычислим, что

|  |  |  |
| --- | --- | --- |
| *Q* | = | 4 |

 и

|  |  |  |
| --- | --- | --- |
| *R* | = | −1.5 |

.  
Далее по формуле

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *S* | = | |  |  | | --- | --- | | *Q* | 3 | | − | |  |  | | --- | --- | | *R* | 2 | |  |

 видим, что

|  |  |  |
| --- | --- | --- |
| *S* | > | 0, |

поэтому уравнение будет иметь три вещественных корня.  
Которые вычисляются по следующим формулам:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | *x* | 1 | | = | −2 | |  |  | | --- | --- | | √ | *Q* | | cos | |  |  |  | | --- | --- | --- | | ( | *ψ* | ) | | − | |  | | --- | | *a* | | 3 | | | | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | *x* | 2 | | = | −2 | |  |  | | --- | --- | | √ | *Q* | | cos | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | ( | *ψ* | + | |  | | --- | | 2 | | 3 | | *π* | ) | | − | |  | | --- | | *a* | | 3 | | | | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | *x* | 3 | | = | −2 | |  |  | | --- | --- | | √ | *Q* | | cos | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | ( | *ψ* | − | |  | | --- | | 2 | | 3 | | *π* | ) | | − | |  | | --- | | *a* | | 3 | | | | | | | |
| где *ψ* | = | |  | | --- | | 1 | | 3 | | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | | (arccos | |  | | --- | | *R* | | |  |  |  |  | | --- | --- | --- | --- | | √ | |  |  | | --- | --- | | *Q* | 3 | | | | ) | | |

.  
Подставив наши числа в эти формулы, мы получим:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | *x* | 1 | | = | −3.3316 | ; | |  |  | | --- | --- | | *x* | 2 | | = | 3.5829 | ; | |  |  | | --- | --- | | *x* | 3 | | = | −0.2513. |

4) Стационарные точки , интервалы возрастания и убывания функции , экстремумы функции

Исследуем функцию на экстремумы и монотонность. Для этого найдем первую производную функции: y’ = (x3 – 12x - 3)’ = 3x2 – 12 = 3(x2 – 4).

Приравняем первую производную к нулю и найдем стационарные точки (в которых y′=0): 3(x2 – 4) = 0, x = ±2.

Получили две критических точки:  х = -2 и х = 2.

Разобьем всю область определения функции на интервалы данными точками и определим знаки производной в каждом промежутке:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x = | -3 | -2 | 0 | 2 | 3 |
| y' = | 15 | 0 | -12 | 0 | 15 |

При x ∈ (−2; 2) производная y′ < 0, поэтому функция убывает на данном промежутке.

При x ∈ (-∞; -2) U (2; ∞) производная y′ > 0, функция возрастает на данных промежутках. При этом x = -2 - точка локального максимума (функция возрастает, а потом убывает, x = 2 - точка локального минимума (функция убывает, а потом возрастает.

Значение функции в этих точках: у(-1) = 13, у(1) = -19.

5) Дополнительные точки для построения графика функции y(x) = x3 − 12x - 3:

|  |  |
| --- | --- |
| **x** | **y** |
| -4.0 | -19 |
| -3.5 | -3.9 |
| -3.0 | 6 |
| -2.5 | 11.4 |
| -2.0 | 13 |
| -1.5 | 11.6 |
| -1.0 | 8 |
| -0.5 | 2.9 |
| 0 | -3 |
| 0.5 | -8.9 |
| 1.0 | -14 |
| 1.5 | -17.6 |
| 2.0 | -19 |
| 2.5 | -17.4 |
| 3.0 | -12 |
| 3.5 | -2.1 |
| 4.0 | 13 |
| 4.5 | 34.1 |

6) По полученным данным строим график, и отметим характерные точки (пересечения с осями и экстремумы).

