

$$x^2(1+x)^2 + x^2 = 8(1+x)^2$$

$$x^2(1+x)^2 + x^2 - 8(1+x)^2 = 0$$

$$x^2(x+1)^2 + x^2 - 8(x+1)^2 = 0$$

$$(x^2(x+1)^2 + x^2) - 8(x+1)^2 = 0$$

$$(x^2(x^2 + 2x + 1) + x^2) - 8(x+1)^2 = 0$$

$$(x^4 + 2x^3 + 2x^2) - 8(x+1)^2 = 0$$

$$(x^4 + (2x^2)(x+1)) - 8(x+1)^2 = 0$$

$$x^4 + (2x^2)(x+1) - 8(x+1)^2 = 0$$

Пусть $A = x^2$; $B = x + 1$

$$A^2 + 2AB - 8B^2 = 0$$

$$A^2 - 2AB + 4AB - 8B^2 = 0$$

$$(A^2 - 2AB) + (4AB - 8B^2) = 0$$

$$(A - 2B)A + (A - 2B)(4B) = 0$$

$$(A - 2B)(A + 4B) = 0$$

$$(x^2 - 2(x+1))(x^2 + 4(x+1)) = 0$$

Случай 1.

$$x^2 - 2(x+1) = 0$$

$$x^2 - 2x - 2 = 0$$

$$D = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-2) = 12$$

$$x_1 = \frac{2 - 2\sqrt{3}}{2 \cdot 1} = 1 - \sqrt{3}; x_2 = \frac{2 + 2\sqrt{3}}{2 \cdot 1} = 1 + \sqrt{3}$$

Случай 2.

$$x^2 + 4(x+1) = 0$$

$$x^2 + 4x + 4 = 0$$

$$D = b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot 4 = 0$$

$$x_{1,2} = -\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2$$

$$\text{Ответ: } x = -2; x = 1 - \sqrt{3}; x = 1 + \sqrt{3}$$