

$$\cos^2 x - 0,5 \sin x > 1$$

$$1 - \cos^2 x + 0,5 \sin x < 0 \Rightarrow \sin^2 x + 0,5 \sin x < 0 \Rightarrow$$

$$\sin x (\sin x + 0,5) < 0$$



$$-0,5 < \sin x < 0$$

$$(-1)^{k+1} \frac{\pi}{6} + \pi k < x < \pi k \quad k \in \mathbb{Z}$$

Пользуясь случаем, даю вывод формулы

$$\cos x \pm \sin x$$

$$\begin{aligned} 1) \cos x + \sin x &= \cos x + \cos\left(\frac{\pi}{2} - x\right) = 2 \cos \frac{\frac{\pi}{2} - x + x}{2} \cos \frac{x - \frac{\pi}{2} + x}{2} = \\ &= 2 \cos \frac{\pi}{4} \cdot \cos\left(x - \frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} \cdot \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2} \cos\left(\frac{\pi}{4} - x\right), \text{ т.к.} \end{aligned}$$

$$\cos(-x) = \cos x$$

$$\begin{aligned} 2) \cos x - \sin x &= \sin\left(\frac{\pi}{2} + x\right) - \sin x = 2 \cos \frac{\frac{\pi}{2} + x + x}{2} \sin \frac{x + \frac{\pi}{2} - x}{2} = \\ &= 2 \cos\left(\frac{\pi}{4} + x\right) \cdot \sin \frac{\pi}{4} = \sqrt{2} \cos\left(\frac{\pi}{4} + x\right) \end{aligned}$$

$$\begin{aligned} 3) \cos x + \sin x &= \sin\left(\frac{\pi}{2} + x\right) + \sin x = 2 \sin \frac{\frac{\pi}{2} + x + x}{2} \cos \frac{\frac{\pi}{2} + x - x}{2} = \\ &= 2 \sin\left(\frac{\pi}{4} + x\right) \cos \frac{\pi}{4} = \sqrt{2} \sin\left(\frac{\pi}{4} + x\right) \end{aligned}$$

$$\begin{aligned} 4) \cos x - \sin x &= \cos x - \cos\left(\frac{\pi}{2} - x\right) = 2 \cos \frac{x + \frac{\pi}{2} - x}{2} \sin \frac{\frac{\pi}{2} - x - x}{2} = \\ &= 2 \cos \frac{\pi}{4} \sin\left(\frac{\pi}{4} - x\right) \end{aligned}$$

Объединим 1) и 2)

$$\cos x \pm \sin x = \sqrt{2} \cos\left(\frac{\pi}{4} \mp x\right)$$

Объединим 3) и 4)

$$\cos x \pm \sin x = \sqrt{2} \sin\left(\frac{\pi}{4} \pm x\right)$$

Теперь понятно?