Запишем матрицу в виде:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 2 | 3 | | 2 | -1 | 3 | | 3 | 3 | -4 | |  | |  |

Главный определитель  
∆=-1\*((-1)\*(-4) - 3\*3) - 2\*(2\*(-4) - 3\*3) + 3\*(2\*3 - (-1)\*3) = 66.  
Определитель отличен от нуля, следовательно, матрица является невырожденной и для нее можно найти обратную матрицу A-1.  
Обратная матрица будет иметь следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d=\frac%7b1%7d%7b66%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | A11 | A21 | A31 | | A12 | A22 | A32 | | A13 | A23 | A33 | |  | |  |

где Aij - алгебраические дополнения.  
**Транспонированная матрица**.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 2 | 3 | | 2 | -1 | 3 | | 3 | 3 | -4 | |  | |  |

Найдем **алгебраические дополнения** матрицы AT.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A1,1 = (-1)1+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 3 | | 3 | -4 | |  | |  |

∆1,1 = ((-1)\*(-4) - 3\*3) = -5

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A1,2 = (-1)1+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 3 | | 3 | -4 | |  | |  |

∆1,2 = -(2\*(-4) - 3\*3) = 17

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A1,3 = (-1)1+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | -1 | | 3 | 3 | |  | |  |

∆1,3 = (2\*3 - 3\*(-1)) = 9

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A2,1 = (-1)2+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 3 | | 3 | -4 | |  | |  |

∆2,1 = -(2\*(-4) - 3\*3) = 17

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A2,2 = (-1)2+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 3 | | 3 | -4 | |  | |  |

∆2,2 = ((-1)\*(-4) - 3\*3) = -5

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A2,3 = (-1)2+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 2 | | 3 | 3 | |  | |  |

∆2,3 = -((-1)\*3 - 3\*2) = 9

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A3,1 = (-1)3+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 3 | | -1 | 3 | |  | |  |

∆3,1 = (2\*3 - (-1)\*3) = 9

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A3,2 = (-1)3+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 3 | | 2 | 3 | |  | |  |

∆3,2 = -((-1)\*3 - 2\*3) = 9

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A3,3 = (-1)3+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 2 | | 2 | -1 | |  | |  |

∆3,3 = ((-1)\*(-1) - 2\*2) = -3  
**Обратная матрица**.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d=\frac%7b1%7d%7b66%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -5 | 17 | 9 | | 17 | -5 | 9 | | 9 | 9 | -3 | |  | |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A-1=** | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | **-5/66** | **17/66** | **3/22** | | **17/66** | **-5/66** | **3/22** | | **3/22** | **3/22** | **-1/22** | |  | |  |

Проверим правильность нахождения обратной матрицы путем умножения исходной матрицы на обратную. Должны получить единичную матрицу *E*.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| E=A\*A-1= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 2 | 3 | | 2 | -1 | 3 | | 3 | 3 | -4 | |  | | https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b1%7d%7b66%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -5 | 17 | 9 | | 17 | -5 | 9 | | 9 | 9 | -3 | |  | |  |

E=A\*A-1=

|  |  |  |
| --- | --- | --- |
| (-1)\*(-5)+2\*17+3\*9 | (-1)\*17+2\*(-5)+3\*9 | (-1)\*9+2\*9+3\*(-3) |
| 2\*(-5)+(-1)\*17+3\*9 | 2\*17+(-1)\*(-5)+3\*9 | 2\*9+(-1)\*9+3\*(-3) |
| 3\*(-5)+3\*17+(-4)\*9 | 3\*17+3\*(-5)+(-4)\*9 | 3\*9+3\*9+(-4)\*(-3) |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl==\frac%7b1%7d%7b66%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 66 | 0 | 0 | | 0 | 66 | 0 | | 0 | 0 | 66 | |  | |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A\*A-1= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 1 | 0 | 0 | | 0 | 1 | 0 | | 0 | 0 | 1 | |  | |  |

Решение верно.