1) $\lim\limits_{n\to\infty}\sqrt[n]{2^n+5^n} = \lim\limits_{n\to\infty}(2^n+5n)^{\frac{1}{n}} = \lim\limits_{n\to\infty}\left[2^{\frac{n}{n}}\cdot\left(1+\left(\frac{5}{2}\right)^n\right)^{\frac{1}{n}}\right] =$

$= \lim\limits_{n\to\infty} 2\cdot\lim\limits_{n\to\infty}(1+2{,}5^n)^{\frac{1}{n}} = 2e^{\lim\limits_{n\to\infty}\frac{\ln(1+2{,}5^n)}{n}}$

$\lim\limits_{n\to\infty}\frac{(\ln(1+2{,}5^n))'}{n'} = \lim\limits_{n\to\infty}\frac{2{,}5^n\ln 2{,}5}{1+2{,}5^n} = \ln 2{,}5\cdot\lim\limits_{n\to\infty}\frac{(2{,}5^n)'}{(1+2{,}5^n)'} =$

$= \ln 2{,}5\cdot\lim\limits_{n\to\infty}\frac{2{,}5^n\ln 2{,}5}{2{,}5^n\ln 2{,}5} = \ln 2{,}5$

$2e^{\ln 2{,}5} = 2\cdot 2{,}5 = 5$

2) $\lim\limits_{n\to\infty}\sqrt[n]{4^n-3^n} = \lim\limits_{n\to\infty}(4^n-3^n)^{\frac{1}{n}} = \lim\limits_{n\to\infty}\left[3^{\frac{n}{n}}\left(\left(\frac{4}{3}\right)^n-1\right)^{\frac{1}{n}}\right] =$

$= \lim\limits_{n\to\infty} 3\cdot\lim\limits_{n\to\infty}\left(\left(\frac{4}{3}\right)^n-1\right)^{\frac{1}{n}} = 3e^{\lim\limits_{n\to\infty}\frac{\ln\left(\left(\frac{4}{3}\right)^n-1\right)}{n}}$

$\lim\limits_{n\to\infty}\frac{\left(\ln\left(\left(\frac{4}{3}\right)^n-1\right)\right)'}{n'} = \lim\limits_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n\ln\frac{4}{3}}{\left(\frac{4}{3}\right)^n-1} = \ln\frac{4}{3}\lim\limits_{n\to\infty}\frac{\left(\left(\frac{4}{3}\right)^n\right)'}{\left(\left(\frac{4}{3}\right)^n-1\right)'} =$

$= \ln\frac{4}{3}\lim\limits_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n\ln\frac{4}{3}}{\left(\frac{4}{3}\right)^n\ln\frac{4}{3}} = \ln\frac{4}{3}$

$3e^{\ln\frac{4}{3}} = 3\cdot\frac{4}{3} = 4$

3) $\lim\limits_{n\to\infty}\left(\frac{(n+1)!+(n+2)!}{(n+3)!}\right) = \lim\limits_{n\to\infty}\frac{(n+1)!(1+n+2)}{(n+1)!(n+2)(n+3)} = \lim\limits_{n\to\infty}\frac{n+3}{(n+2)(n+3)} =$

$= \lim\limits_{n\to\infty}\frac{1}{n+2} = 0$

4) $\lim\limits_{n\to\infty}\left(\frac{n^3}{n^2+5}-n\right) = \lim\limits_{n\to\infty}\left(\frac{n^3-n(n^2+5)}{n^2+5}\right) = \lim\limits_{n\to\infty}\frac{n^3-n^3-5n}{n^2+5} =$

$= \lim\limits_{n\to\infty}\frac{-(5n)'}{(n^2+5)'} = -5\lim\limits_{n\to\infty}\frac{1}{2n} = 0$

5) $\lim\limits_{n\to\infty}\left(\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right) = \lim\limits_{n\to\infty}\frac{\left(\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right)\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2}\sqrt[3]{(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}{\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2}\sqrt[3]{(n-1)^2}+\sqrt[3]{(n-1)^4}} =$

$= \lim\limits_{n\to\infty}\frac{n+1-n+1}{\sqrt[3]{(n+1)^4}+\sqrt[3]{(n^2-1)^2}+\sqrt[3]{(n-1)^4}} = \frac{2}{\infty} = 0$

б) $$\lim_{n\to\infty}\frac{2^n-2^{-n}}{2^n+2^{-n}}=\lim_{n\to\infty}\frac{2^n-\frac{1}{2^n}}{2^n+\frac{1}{2^n}}=\lim_{n\to\infty}\frac{2^{2n}-1}{2^{2n}-1}=\left|\frac{\infty}{\infty}\right|=$$

$$=\lim_{n\to\infty}\frac{(4^n-1)'}{(4^n+1)'}=\lim_{n\to\infty}\frac{4^n\ln 4}{4^n\ln 4}=\underline{1}$$